Table of Contents

How to Use This Addenda

Make sure you're ready to teach by noting the **Necessary Materials and Pre-Lesson Prep** you will need to gather or complete prior to the lesson

Find high-leverage instructional moves in the **Lesson Look Fors**. This is what leaders should see when observing your instruction

Note how your lesson objective ties to your state **Standards**

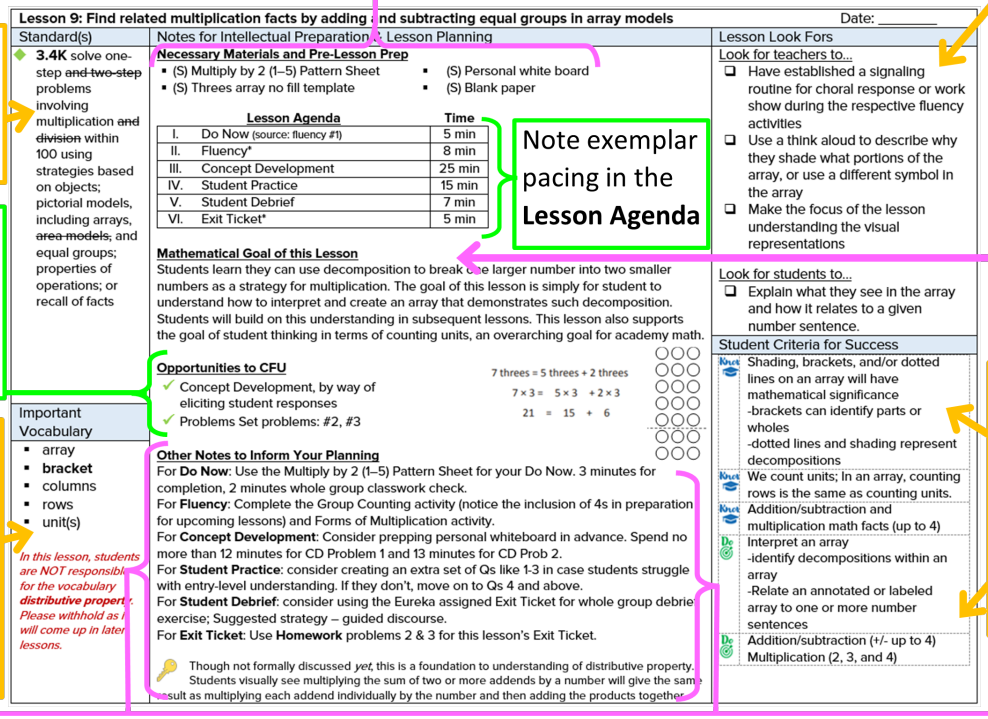
Note exemplar pacing in the **Lesson Agenda**

Use the **Mathematical Goal of the Lesson** to keep you focused on the appropriate student outcome

Plan purposeful questioning and responses using **Opportunities to CFU**

Plan instruction around what students need to Know & Do to be successful on the Exit Ticket using the identified **Student Know/Do Chart**

Plan to stress **Important Vocabulary** in the lesson. New vocab for the unit is indicated in bold



Lesson 9: Find related multiplication facts by adding and subtracting equal groups in array models Date: _____

Standard(s)

3.4K solve one-step ~~and two-step~~ problems involving multiplication ~~and division~~ within 100 using strategies based on objects; pictorial models, including arrays, ~~area models,~~ and equal groups; properties of operations; or recall of facts

Important Vocabulary

- array
- **bracket**
- columns
- rows
- unit(s)

In this lesson, students are NOT responsible for the vocabulary ***distributive property****. Please withhold as it will come up in later lessons.*

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- (S) Multiply by 2 (1–5) Pattern Sheet
- (S) Threes array no fill template
- (S) Personal white board
- (S) Blank paper

Lesson Agenda		Time
I.	Do Now (source: fluency #1)	5 min
II.	Fluency*	8 min
III.	Concept Development	25 min
IV.	Student Practice	15 min
V.	Student Debrief	7 min
VI.	Exit Ticket*	5 min

Mathematical Goal of this Lesson

Students learn they can use decomposition to break one larger number into two smaller numbers as a strategy for multiplication. The goal of this lesson is simply for student to understand how to interpret and create an array that demonstrates such decomposition. Students will build on this understanding in subsequent lessons. This lesson also supports the goal of student thinking in terms of counting units, an overarching goal for academy math.

Opportunities to CFU

- ✓ Concept Development, by way of eliciting student responses
- ✓ Problems Set problems: #2, #3

7 threes = 5 threes + 2 threes
7 × 3 = 5 × 3 + 2 × 3
21 = 15 + 6

Other Notes to Inform Your Planning

For **Do Now**: Use the Multiply by 2 (1–5) Pattern Sheet for your Do Now. 3 minutes for completion, 2 minutes whole group classwork check.
For **Fluency**: Complete the Group Counting activity (notice the inclusion of 4s in preparation for upcoming lessons) and Forms of Multiplication activity.
For **Concept Development**: Consider prepping personal whiteboard in advance. Spend no more than 12 minutes for CD Problem 1 and 13 minutes for CD Prob 2.
For **Student Practice**: consider creating an extra set of Qs like 1-3 in case students struggle with entry-level understanding. If they don't, move on to Qs 4 and above.
For **Student Debrief**: consider using the Eureka assigned Exit Ticket for whole group debrief exercise; Suggested strategy – guided discourse.
For **Exit Ticket**: Use **Homework** problems 2 & 3 for this lesson's Exit Ticket.

Though not formally discussed *yet*, this is a foundation to understanding of distributive property. Students visually see multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together.

Lesson Look Fors

Look for teachers to...

- ❑ Have established a signaling routine for choral response or work show during the respective fluency activities
- ❑ Use a think aloud to describe why they shade what portions of the array, or use a different symbol in the array
- ❑ Make the focus of the lesson understanding the visual representations

Look for students to...

- ❑ Explain what they see in the array and how it relates to a given number sentence.

Student Criteria for Success

- Know: Shading, brackets, and/or dotted lines on an array will have mathematical significance -brackets can identify parts or wholes -dotted lines and shading represent decompositions
- Know: We count units; In an array, counting rows is the same as counting units.
- Know: Addition/subtraction and multiplication math facts (up to 4)
- Do: Interpret an array -identify decompositions within an array -Relate an annotated or labeled array to one or more number sentences
- Do: Addition/subtraction (+/- up to 4) Multiplication (2, 3, and 4)

Find recommended lesson modifications, content knowledge boosters, and/or high-leverage instructional moves that may not be in your Teacher Edition located in **Other Notes to Inform Your Planning**

UNIT SYNOPSIS

In Unit 1, we look at a special type of equation involving a dependent variable and independent variable called a function. This unit will define what a function is, how to graph functions, discuss properties of functions, and contextualizing functions in real-life settings. We will extend these discussions to other types of functions and relationships, such as composition of functions, even and odd functions, transformations of functions, inverse functions and/or relations, and interpreting mathematical models.

This unit features topics involving domain, range, transformations on functions, symmetry, local and extreme behavior, composition of functions, and one-to-one and inverse functions.

CONTENT STANDARDS

Below are the standards addressed in this unit.

Texas Essential Knowledge and Skills (TEKS)	
Knowledge and Skills	**Student Expectations (SE)**
(2) Functions The student uses process standards in mathematics to **explore**, **describe**, and **analyze** the **attributes of functions**. The student makes connections between **multiple representations** of functions and **algebraically constructs** new functions. The student analyzes and uses functions to **model** real-world problems.	**(2.A)** Use the composition of two functions to model and solve real-world problems. **(2.B)** Demonstrate that function composition is not always commutative. **(2.C)** Represent a given function as a composite function of two or more functions. **(2.D)** Describe symmetry of graphs of even and odd functions. **(2.E)** Determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse using multiple representations. **(2.F)** Graph ~~exponential~~, ~~logarithmic~~, ~~rational~~, polynomial, power, ~~trigonometric~~, ~~inverse trigonometric~~, and piecewise defined functions, including step functions. **(2.G)** Graph functions, including ~~exponential~~, ~~logarithmic~~, ~~sine~~, ~~cosine~~, ~~rational~~, polynomial, and power functions and their transformations, including $af(x)$, $f(x)+d$, $f(x-c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems. **(2.I)** Determine and analyze the key features of ~~exponential~~, ~~logarithmic~~, ~~rational~~, polynomial, power, ~~trigonometric~~, ~~inverse trigonometric~~, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing.

[2]Parts of standard that are crossed out are not taught in this unit but will be taught in future units.

Focus on Disciplinary Literacy 	Mathematical Process Standard (F) – Analyze mathematical relationships to connect and communicate mathematical ideas.
	Mathematical Process Standard (G) – Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

LEARNING SUPPORTS BY LESSON

There is a checkmark for the math support if the lesson	Lessons → Math Supports	L1	L2	L3	L4	L5	L6	L7	L8	L9
makes a connection to prior content or from a previous unit or academic year	Access Prior Knowledge	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses familiar contexts or experiences to make the learning relevant to students	Real-World Connections				✓			✓	✓	✓
makes use of graphic organizers	Graphic Organizers	✓	✓	✓	✓	✓	✓	✓	✓	✓
includes tools like rulers, protractors, patty paper, algebra tiles, etc.	Tools or Manipulatives									
incorporates tables, reference charts, displays, pictures, models, or color-coding	Visual Aids	✓	✓	✓	✓	✓	✓	✓	✓	✓
includes definitions, examples vs. nonexamples, cognates, etc.	Vocabulary Supports	✓	✓	✓	✓	✓	✓	✓	✓	✓
includes strategies that support language development										
asks students to discuss with their partner to prepare for whole class discussion	- Turn and Talk	✓	✓	✓	✓	✓	✓	✓	✓	✓
teacher facilitates a whole class discussion to debrief key learnings	- Guided Discussion	✓	✓	✓	✓	✓	✓	✓	✓	✓
asks students to think independently, test their idea with a partner, and share whole group	- Think, Pair, Share			✓	✓	✓	✓	✓	✓	✓
includes sentence stems to support students with explanations	- Sentence Stems	✓						✓		✓
provides opportunities for students to work with a partner or a group	Peer Collaboration	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses mnemonics such as SohCahToa	Mnemonics			✓						
includes websites or equipment that enhances the lesson	Technological Support		✓	✓	✓	✓	✓	✓	✓	✓
content can be presented in different forms										
uses hands-on tools or manipulatives to represent the math	- Concrete									
uses drawings to represent the math	- Pictorial	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses numbers and number sentences to represent the math	- Abstract	✓	✓	✓	✓	✓	✓	✓	✓	✓

ROADMAP

AT A GLANCE: Unit 1 - Foundations of Functions			
Day	Date	Lesson	Lesson Title
1		1	Function Notation, Domain, and Range
2		2	Transformations of Functions
3		3	Applying Transformations
4		4	Evaluating Piecewise Functions
5		5	Graphing Piecewise Functions
6		6	Even/Odd Symmetry
7		7	Increasing, Decreasing, Boundedness, and Extrema
8		8	Composition of Functions
9		9	Inverse and One-to-One Functions
10			*Unit 1 Success Day Alpha – Applications of Functions (MCR Optional Lesson)*
11			*Unit 1 Success Day Beta– Unit Review or Flex Day as needed based on your data*
12			End of Unit 1 Assessment

Date: _____

Lesson 1: Function Notation, Domain, and Range

Standard(s)

- **(2.I)** Determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, ~~trigonometric, inverse trigonometric~~, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing.
- **(2.L)** Determine various types of discontinuities in the interval $(-\infty, \infty)$ as they relate to functions.

Important Vocabulary

- **Function Notation**
- **Radical Function**
- **Rational Function**
- **Domain (inputs)**
- **Denominator Restriction**
- **Square Root (even root) Restriction**
- **Range (outputs)**
- **Interval Notation**
- **U ("union")**
- **Point of discontinuity**
- **Continuous graph**
- **Vertical asymptote**
- **Horizontal asymptote**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Desmos
- Pre-cut card sort activity, *including number needed per student or pair of students*

Lesson Structure:

- Do Now (7 min)
- INM (22 min)
- Student Practice (18 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to state the domain and range of a function using set and interval notation as well as build fluency in functional notation.

Opportunities to CFU

- ✓ Domain vs Range (INM)
- ✓ When do rational functions have a discontinuity? (INM)
- ✓ When do radical functions have "gap" in the domain? (INM)
- ✓ How do you join two intervals? (Debrief)

Other Notes to Inform Your Planning

Focus on Disciplinary Literacy

INM & Debrief

- By the end of the first quarter, students will be able to assess domain, range, and some aspects of continuity for polynomial (Unit 2), rational (Unit 2), exponential (Unit 3), logarithmic (Unit 3), piecewise defined functions, and trigonometric functions (Unit 4).
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2012 AP Calculus AB released exam.

11. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x. Which of the following statements is true?

(A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\lim_{x \to 2} f(x) \neq 0$

(E) $x = 2$ is a vertical asymptote of the graph of f.

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking.

Look for students to…

- ❑ Determine why certain values of x cannot be evaluate for specific functions.
- ❑ Explain why the domain of a function may include an open interval.

Student Know/Do Chart

Know	The set of all possible input values of a function is called the **domain**, and the set of all possible output values of a functions is called the **range**.
Know	Breaks and gaps in the domain of a function occur when it is not possible to evaluate the function for certain input values.
Do	Determine the domain and range of a function and when it is appropriate to use open or closed intervals.
Do	Articulate where rational and radical functions contain a break or gap in the domain.

Date: _____

Lesson 2: Transformations of Functions

Standard(s)

- **(2.F)** Graph exponential, logarithmic, rational, polynomial, power, ~~trigonometric~~, ~~inverse trigonometric~~, and piecewise defined functions, including step functions.
- **(2.G)** Graph functions, including exponential, logarithmic, ~~sine, cosine~~, rational, polynomial, and power functions and their transformations, including $af(x)$, $f(x)+d$, $f(x-c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems.

Important Vocabulary

- **Vertical stretch/shrink**
- **Horizontal stretch/shrink**
- **Horizontal translation left/right**
- **Vertical translation up/down**
- **Reflection across the x-axis**
- **Reflection across the y-axis**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Desmos
- Chart paper; *optional, but useful to post on your wall as a reference.*

Lesson Structure:

- Do Now (6 min)
- INM (23 min)
- Student Practice (18 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will investigate and summarize the effects of a parent function when adding parameters $a, b, c,$ and d to the expression $a \cdot f\big(b(x+c)\big)+d$ using technology.

Opportunities to CFU

- ✓ How does shape and scale of the original graph compare to the modified version? (Do Now)
- ✓ When do rational functions have a discontinuity? (INM)
- ✓ What is the difference between $f(x+c)$ and $f(x)+d$? (INM)
- ✓ What is the difference between $a \cdot f(x)$ and $f(bx)$? (INM)

Other Notes to Inform Your Planning

Focus on Disciplinary Literacy

INM & Debrief

- In this unit, students will be able to generalize the parameters that affect the shape and location of a function from its parent function. Throughout Semester 1, students will revisit transformations and apply them to different families of function, such as exponential, logarithmic, and sinusoidal functions.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2008 AP Calculus AB released exam.

77. If $\int_0^3 f(x)\,dx = 6$ and $\int_3^5 f(x)\,dx = 4$, then $\int_0^5 (3+2f(x))\,dx =$

(A) 10 (B) 20 (C) 23 (D) 35 (E) 50

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking.
- ❑ Uses Desmos in class as a tool to visualize the effects of the parameters acting on a parent function.
  - *Teacher projects on the board or students use their assigned tech.*

Look for students to…

- ❑ Explain how a and b are similar and different.
- ❑ Explain how a and b are similar and different.
- ❑ Explain why certain values of a and b reflect about the x- or y-axis.

Student Know/Do Chart

Know — Parameters a and b determine the shape of the transformed function.

Know — Parameters c and d determine the location of the transformed function.

Do — Match the equation of a function in *vertex* form with its corresponding graph, and vice versa.

Do — Write the equation of a parent function when specified parameters $a, b, c,$ and d are applied, and vice versa.

Date: _____

Lesson 3: Applying Transformations

Standard(s)

- **(2.G)** Graph functions, including exponential, logarithmic, ~~sine, cosine~~, rational, polynomial, and power functions and their transformations, including $af(x)$, $f(x)+d$, $f(x-c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems.

Important Vocabulary

- Vertical stretch/shrink
- Horizontal stretch/shrink
- Horizontal translation left/right
- Vertical translation up/down
- Reflection across the x-axis
- Reflection across the y-axis
- **Rigid transformations**
- **nonrigid transformations**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Desmos
- Posters and markers; *for each group or pair of students during the Student Practice.*

Lesson Structure:

- Do Now (6 min)
- INM (22 min)
- Student Practice (19 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

Today's lesson is a deeper examination into transformations of functions, with a focus on how the transformations adjust the actual coordinate points, and what happens when a function has multiple transformations.

Opportunities to CFU

- ✓ What affect does $f(-x)$ and $-f(x)$ have on the coordinate $(x, f(x))$? (INM)
- ✓ How can we determine the proper order in which we apply multiple transformations to a function simultaneously? (End of INM)

Other Notes to Inform Your Planning

Focus on Disciplinary Literacy

INM & Debrief

- In this unit, students will be able to generalize the parameters that affect the shape and location of a function from its parent function. Throughout Semester 1, students will revisit transformations and apply them to different families of function, such as exponential, logarithmic, and sinusoidal functions.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2008 AP Calculus AB released exam.

77. If $\int_0^3 f(x)\,dx = 6$ and $\int_3^5 f(x)\,dx = 4$, then $\int_0^5 (3 + 2f(x))\,dx =$

(A) 10 (B) 20 (C) 23 (D) 35 (E) 50

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking.
- ❑ Uses Desmos in class as a tool to visualize the effects of the parameters acting on a parent function.
  - *Teacher projects on the board or students use their assigned tech.*

Look for students to…

- ❑ Explain how a and b are similar and different.
- ❑ Explain how a and b are similar and different.
- ❑ Explain why certain values of a and b reflect about the x- or y-axis.

Student Know/Do Chart

Know: Parameters a and b determine the shape of the transformed function.

Know: Parameters c and d determine the location of the transformed function.

Do: Match the equation of a function in *vertex* form with its corresponding graph, and vice versa.

Do: Write the equation of a parent function when specified parameters a, b, c, and d are applied, and vice versa.

Date: _____

Lesson 4: Evaluating Piecewise Functions

Standard(s)

◆ **(2.F)** Graph ~~exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric~~, and piecewise defined functions, including step functions.

Important Vocabulary

- **Piecewise function**
- **Evaluate**
- **Vertical line test**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Desmos

Lesson Structure:

- Do Now (7 min)
- INM (20 min)
- Student Practice (20 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to write, evaluate, and analyze key characteristics piecewise-defined functions, such as domain and continuity; and apply then in contextual setting.

Opportunities to CFU

- ✓ How can you assess the continuity of a piecewise-defined function? (INM)
- ✓ How are piecewise functions characterized? (INM)

Other Notes to Inform Your Planning

- The Do Now serves as a primer for the lesson where students can connect the concept of multiple functions applied to a contextual setting.
- Not all the time can we model natural phenomena using a function. At times, we can describe models using multiple functions over a continuous domain. This does not imply that the piecewise functions in this course will always be continuous.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the **2012 AP Calculus AB released exam**. This is one of *many* settings where piecewise functions can be applied in AP Calculus.

Focus on Disciplinary Literacy

INM & Debrief

13. The function f is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3. \end{cases}$ What is the value of $\int_1^5 f(x)\,dx$?

(A) 2 (B) 6 (C) 8 (D) 10 (E) 12

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking.
- ❑ Unpacking problems involving a scenario by annotating and identifying keywords in the text.

Look for students to…

- ❑ Explain why the subfunctions of a piecewise function contain restricted domains.
- ❑ Determine which restricted domains contain open or closed endpoints.
- ❑ Explain how to evaluate a piecewise function when given specified input values.

Student Know/Do Chart

Know — A piecewise function is composed of "subfunctions" each with their corresponding restricted domains.

Know — The graphs of piecewise function may or may not be continuous.

Do — Construct piecewise-defined functions given the context of a situation including restricted domains.

Do — Assess the continuity of a piecewise function.

Do — Evaluate piecewise functions for specified input values.

Date: _____

Lesson 5: Graphing Piecewise Functions

Standard(s)

- **(2.F)** Graph ~~exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric~~, and piecewise defined functions, including step functions.

Important Vocabulary

- Piecewise function
- Evaluate
- Vertical line test

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Chart paper; *optional, but useful to post on your wall as a reference.*

Lesson Structure:

- Do Now (6 min)
- INM (21 min)
- Student Practice (20 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In Lesson 1.4, students investigated evaluating piecewise functions algebraically and graphically. In this lesson, students will focus on graphing piecewise functions.

Opportunities to CFU

- ✓ How does $<$ and $\leq$ differ when graphing piecewise functions? (INM)
- ✓ How is graphing a piecewise function similar to graphing a regular function? How is it different? (Debrief)

Other Notes to Inform Your Planning

- The Do Now serves as a review of the previous lesson. Students will expand on this and apply transformations for each subfunction in the INM.
- Not all the time can we model natural phenomena using a function. At times, we can describe models using multiple functions over a continuous domain. This does not imply that the piecewise functions in this course will always be continuous.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2008 AP Calculus AB released exam.

Focus on Disciplinary Literacy

INM & Debrief

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking.
- ❑ Unpacking problems involving a scenario by annotating and identifying keywords in the text.

Look for students to…

- ❑ Explain why the subfunctions of a piecewise function contain restricted domains.
- ❑ Determine which restricted domains contain open or closed endpoints.
- ❑ Explain how to evaluate a piecewise function when given specified input values.

Student Know/Do Chart

Know — A piecewise function is composed of "subfunctions" each with their corresponding restricted domains.

Know — The graphs of piecewise function may or may not be continuous.

Do — Construct piecewise-defined functions given the context of a situation including restricted domains.

Do — Assess the continuity of a piecewise function.

Do — Evaluate piecewise functions for specified input values.

Date: _____

Lesson 6: Even/Odd Symmetry

Standard(s)

- **(2.D)** Describe symmetry of graphs of even and odd functions.

Important Vocabulary

- **Symmetry with respect to:**
  - **x-axis**
  - **y-axis**
  - **origin**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Chart paper; *optional, but useful to post on your wall as a reference.*

Lesson Structure:

- Do Now (6 min)
- INM (21 min)
- Student Practice (20 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be using their prior knowledge of reflections in their investigation of the symmetry of a function. Students will be able to assess even or odd symmetry using $f(-x)$ and $-f(x)$ properties.

Opportunities to CFU

- ✓ How can you determine whether a function is even or odd by looking at its' graph?
- ✓ How can you determine whether a function is even or odd by looking at its' equation?

Other Notes to Inform Your Planning

- The Do Now revisits transformations of function and bridges the concept of how certain reflections and rotations can classify a function as even, odd, or neither.
- Students will spend the time analyzing functions algebraically and graphically to determine if they are even, odd, or neither. Students will also look at how basic transformations affect a function that was even or odd, then they will generalize this into a rule for all transformations of functions.

Focus on Disciplinary Literacy

INM & Debrief

Which function below is an **ODD** function?

a. $f(x) = x^3 + x^2$
b. $f(x) = 4x^3 + 2x$
c. $f(x) = 2x^2 - 8x$
d. $f(x) = 3x^4 + 5x^2$

A function $f(x)$ is an **EVEN** function. If $f(4) = 2$, which other point must be on the graph of $f(x)$?

a. $(4,-2)$
b. $(-4,2)$
c. $(-4,-2)$
d. $(-2,-4)$

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking.

Look for students to…

- ❑ Explain how a negative sign in $-f(x)$ and $f(-x)$ affects the function.
- ❑ Compare the outputs of the original function with $-f(x)$ and $f(-x)$ to determine the if $f(x)$ can be classified as even, odd, or neither.

Student Know/Do Chart

Know	An even function satisfies the property $f(x) = f(-x)$.
Know	An odd function satisfied the property $-f(x) = f(-x)$.
Do	Determine whether a function is even, odd, or neither using its graph.
Do	Determine whether a function is even, odd, or neither algebraically given its equation.

Date: _____

Lesson 7: Increasing, Decreasing, Boundedness, and Extrema

Standard(s)

- **(2.I)** Determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, ~~inverse trigonometric~~, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing.

Important Vocabulary

- **Increasing**
- **Decreasing**
- **Boundedness**
- **Extrema**
- **Maximum**
- **Minimum**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Desmos
- Chart paper; *optional, but useful to post on your wall as a reference.*

Lesson Structure:

- Do Now (6 min)
- INM (18 min)
- Student Practice (25 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will strengthen their ability to discuss characteristics of functions. The terminology will include intervals of increasing and decreasing, local and absolute extrema, and boundedness.

Opportunities to CFU

- ✓ How do we denote when a function increases or decreases? (INM)
- ✓ What is the difference between local and absolute extrema? (INM)
- ✓ Can a function have more than one absolute max? Absolute min? (Debrief)

Other Notes to Inform Your Planning

- Students will revisit these concepts in subsequent units for polynomials, rational functions, and the transcendental functions.
- In AP Calculus, students learn to differentiate functions to determine increasing and decreasing interval and apply the First and Second Derivative Test as well as the Candidates Test to determine extreme function values.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the **2008 AP Calculus AB released exam**.

Focus on Disciplinary Literacy

INM & Debrief

2. Let f be the function given by $f(x) = 300x - x^3$. On which of the following intervals is the function f increasing?

(A) $(-\infty, -10]$ and $[10, \infty)$

(B) $[-10, 10]$

(C) $[0, 10]$ only

(D) $[0, 10\sqrt{3}]$ only

(E) $[0, \infty)$

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking.

Look for students to…

- ❑ Explain the difference between absolute and relative extrema.
- ❑ Distinguish between bounded above, bounded below, and bounded.

Student Know/Do Chart

- Know: Local extrema occurs when a function changes from decreasing to increasing and vice versa.
- Know: In some cases, local extrema can also be absolute extrema.
- Know: Boundedness and absolute extrema are connected.
- Do: Determine local and absolute extrema via a graph.
- Do: Determine boundedness via a graph.

Date: _____

Lesson 8: Composition of Functions

Standard(s)

- **(2.A)** Use the composition of two functions to model and solve real-world problems.
- **(2.B)** Demonstrate that function composition is not always commutative.
- **(2.C)** Represent a given function as a composite function of two or more functions.

Important Vocabulary

- **Composition of functions**
- **Not commutative**
- **Restricted domain**
- **Decomposition**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Show this video during the Do Now

Lesson Structure:

- Do Now (7 min)
- INM (20 min)
- Student Practice (20 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will investigate composition of functions, or the creation of a function made of other functions, where the output of one function is the input to another and use it to evaluate composition of functions.

Opportunities to CFU

- ✓ How do you compose two functions? (INM)
- ✓ Is composition of functions commutative? (INM)

Other Notes to Inform Your Planning

- The Do Now primes students thinking on how function machines can be extended to work as a composition of both machines.
- In the INM, students will explore different representations of evaluating composition functions, such as tables and graphs.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this free-response problem from a recent AP exam.

Focus on Disciplinary Literacy

INM & Debrief

Lesson Look Fors

Look for teachers to…

- ❑ Shows video in the Do Now.
- ❑ Guided discourse, probing questions, and pushing students' thinking.

Look for students to…

- ❑ Explain what type of real scenarios relate to composition of functions.
- ❑ Interpret and write in composition function notation.

Student Know/Do Chart

- Know: Composition of function requires at least two function, one composed of another.
- Know: Evaluating composition of functions has order.
- Do: Evaluate composition functions for specified values of x given two or more functions.
- Do: Evaluate composition function for specified values of x given a table of values.

Date: _____

Lesson 9: Inverse and One-to-One Functions

Standard(s)

- **(2.E)** Determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse using multiple representations.

Important Vocabulary

- **Inverse relation**
- **Horizontal line test**
- **One-to-one**
- **Inverse function**
- **Inverse composition rule**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Word Cloud
- Slide Deck (Stations)

Lesson Structure:

- Do Now (6 min)
- INM (18 min)
- Student Practice (23 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be investigating how to write, graph, and confirm inverse functions.

Opportunities to CFU

- ✓ What does the composition of two inverses produce? (INM)
- ✓ How can you determine if a function has an inverse? (INM)
- ✓ How can an inverse function be found algebraically? Graphically? (INM)

Other Notes to Inform Your Planning

- In the Do Now, students start by explaining what a function is, and solving some basic equations. This prepares them to find the inverse of a function algebraically.
- In the INM, students will see that inverse operations in the reverse order (as in the last machine must be undone first) will produce the input put into the original function.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2008 AP Calculus AB released exam.

Focus on Disciplinary Literacy

INM & Debrief

90. The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$. If $f(3) = 8$ and $f'(3) = 9$, what are the values of $g(8)$ and $g'(8)$?

(A) $g(8) = \frac{1}{3}$ and $g'(8) = -\frac{1}{9}$

(B) $g(8) = \frac{1}{3}$ and $g'(8) = \frac{1}{9}$

(C) $g(8) = 3$ and $g'(8) = -9$

(D) $g(8) = 3$ and $g'(8) = -\frac{1}{9}$

(E) $g(8) = 3$ and $g'(8) = \frac{1}{9}$

Lesson Look Fors

Look for teachers to…

- ❑ Use the Word Wall in the Do Now.
- ❑ Guided discourse, probing questions, and pushing students' thinking.

Look for students to…

- ❑ Develop the inverse relationship in given in the function machine during the INM.
- ❑ Articulate the relationship of a function and its inverse via graphically and using composition of functions.

Student Know/Do Chart

Know	The point $(x, f(x))$ on the original function corresponds to $(f(x), x)$ for its inverse.
Know	Every function has an inverse relation, but not all inverse relations are functions.
Do	Determine if a function has an inverse function graphically using the one-to-one.
Do	Prove two functions are inverses using compositions o functions.

UNPACKED STANDARDS

Focus standards for this unit.

Standards Clarification

Standards

(2.I) Determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, ~~inverse trigonometric~~, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing.

Specificity

Concepts:

Parent Functions:

- Exponential: $f(x) = a \cdot b^x$
- Logarithmic: $f(x) = \log_b(x)$
- Rational: $f(x) = \frac{1}{x}$
- Polynomial: $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$
- Power: $f(x) = a \cdot x^b$
- Trigonometric: $f(x) = \sin(x)$ or $f(x) = \cos(x)$
- Inverse Trigonometric $f(x) = \arcsin(x)$ or $f(x) = \arccos(x)$
- Piecewise-defined functions
- Key features
  - Domain (interval notation*)
  - Range (interval notation*)
  - Symmetry (about x, y, origin)
  - Maximum (local vs. absolute*)
  - Minimum (local vs. absolute*)
  - Extrema*
  - Boundedness*
  - Zeros
  - Asymptotes
  - Intervals of increasing (interval notation*)
  - Intervals of decreasing (interval notation*)

Content:

Including, but not limited to:

- Identify the parent function
- Graph the function
- Determine key features of the function
- Analyze the function by the key features

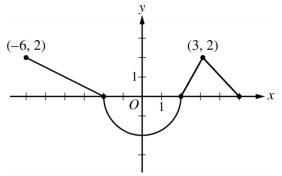
Notes/Explanations/Examples

Below are the content and skill connections between the Algebra 1 TEKS and the AP standard.

Algebra 2 TEKS

(2A.2.A) graph the functions $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b x$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum & minimum given an interval

AP Calculus AB 2017 Released FRQ #3 Parts b & c



Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find the values of $f(-6)$ and $f(5)$.

(b) On what intervals is f increasing? Justify your answer.

(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

(b) $f'(x) > 0$ on the intervals $[-6, -2)$ and $(2, 5)$. Therefore, f is increasing on the intervals $[-6, -2]$ and $[2, 5]$.	2 : answer with justification
(c) The absolute minimum will occur at a critical point where $f'(x) = 0$ or at an endpoint.	2 : $\begin{cases} 1 : \text{considers } x = 2 \\ 1 : \text{answer with justification} \end{cases}$

$f'(x) = 0 \Rightarrow x = -2, x = 2$

x	$f(x)$
-6	3
-2	7
2	$7 - 2\pi$
5	$10 - 2\pi$

The absolute minimum value is $f(2) = 7 - 2\pi$.

VERTICAL STANDARDS

This section details the **progression** of key student expectations/standards** in the courses **before** and **after** this course. This will help you understand what **prior knowledge skills to build upon** and guide you in knowing what **skills you are preparing your students** for in the subsequent course.

Algebra 2	Pre-Calculus	AP Calculus AB Essential Knowledge
• **2A.2A** Graph the functions $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = \lvert x \rvert$, and $f(x) = \log_b x$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum & minimum given an interval. • **2A.2B** Graph and write the inverse of a function using notation such as $f^{-1}(x)$. • **2A.2C** Describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range. • **2A.2D** Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other. • When $f(x)$ is replaced including $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific positive and negative values of a, b, c, and d, determine the effect on the: ○ **2A.4C** Graph of $f(x) = \sqrt{x}$ ○ **2A.5A** Graph of $f(x) = b^x$ and $f(x) = \log_b x$ where b is 2, 10, and e ○ **2A.6A** Graph of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$ ○ **2A.6C** Graph of $f(x) = \lvert x \rvert$ ○ **2A.6G** Graph of $f(x) = \frac{1}{x}$	• **(2.A)** Use the composition of two functions to model and solve real-world problems. • **(2.B)** Demonstrate that function composition is not always commutative. • **(2.C)** Represent a given function as a composite function of two or more functions. • **(2.D)** Describe symmetry of graphs of even and odd functions. • **(2.E)** Determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse using multiple representations. • **(2.F)** Graph exponential, logarithmic, rational, polynomial, power, trigonometric, ~~inverse trigonometric~~, and piecewise defined functions, including step functions. • **(2.G)** Graph functions, including exponential, logarithmic, ~~sine, cosine~~, rational, polynomial, and power functions and their transformations, including $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems. • **(2.I)** Determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, ~~inverse trigonometric~~, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing or decreasing.	• **(LIM-1.D.1)** Limits of sums, differences, products, quotients, and composite functions can be found using limit theorems. • **(LIM-2.B.2)** Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains. • **(FUN-4.A.2)** The first derivative of a function can determine the location of relative (local) extrema of the function. • **(FUN-4.A.3)** Absolute (global) extrema of a function on a closed interval can only occur at critical points or at endpoints. • **(FUN-4.A.7)** The second derivative of a function may determine whether a critical point is the location of a relative (local) maximum or minimum. • **(FUN-4.A.8)** When a continuous function has only one critical point on an interval on its domain and the critical point corresponds to a relative (local) extremum of the function on the interval, then that critical point also corresponds to the absolute (global) extremum of the function on the interval. • **(FUN-7.E.3)** Solutions to differential equations may be subject to domain restrictions. NOTE: These “essential knowledge” (EK) standards are from The College Board Course Exam Description for AP Calculus AB/BC, not the TEKS.